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13 ABSTRACT

Final Scientific Report describing research conducted under this 13-month grant. Seven papers were produced on the subject of applications of the theory of convex functions to optimization problems of various sorts, especially those involving integral functionals.

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Security Classification

Research under this 13-month grant culminated in the writing of four technical papers [2,3,4,5] and two expository papers [1,6], all on applications of convex analysis to problems of optimization. A doctoral dissertation [7] was also produced.

The paper "New Applications of Duality in Convex Programming" [1], written at the beginning of 1971, describes in expository fashion the main ideas and computational motivations behind current work in this area. The discussion is limited to the finite-dimensional case, but the motivation carries over to infinite-dimensional problems, for example in stochastic programming and optimal control. Besides the general and important aim of strengthening the conceptual framework for handling optimization problems of all sorts, such research is directed at the construction of dual problems and minimax problems which may help computationally in solving a given problem. For example, it is now known that to solve a differentiable convex programming problem (with nonlinear constraints), it is enough to be able to minimize a differentiable convex function without constraints, provided one can do this economically in terms of the number of function evaluations. Many results of this type follow from the theory developed under the preceding AFOSR grant. They lead to new speculations about computational possibilities, as indicated in [1]. This paper was presented in invited talks at the Dallas meeting of the Operations Research Society of America (May, 1971) and the Fourth International Conference on Probability (Brașov, Romania, September 1971) and is to appear in the proceedings of the latter.

The most concentrated research effort under this grant has not been on these speculations, however. Rather it has been on strengthening the means for attacking problems of optimal control by the same methods. In a previous paper [11], new tools of convex analysis were employed for the first time to establish some very powerful existence theorems and duality theorems for a broad class of "convex" control problems, leading for example to a generalization of the maximum principle. The time being ripe for applying and augmenting these results, three further papers [3,4,5] were produced during the past year, as well as an expository summary [6].

The paper "State Constraints in Convex Control Problems of Bolza" [3] is the main one of these. It shows how the theory in [11] can be broadened to encompass state constraints, in addition to constraints on controls and terminal points, which could already be handled. The results were presented in an invited lecture at the S.I.A.M. meeting in Seattle, June 1971, and the paper has since been accepted for publication in the SIAM Journal on Control.

In "Optimal Arcs and the Minimum Value Function in Problems of Lagrange" [4], it is demonstrated that the existence theory of [11] for "convex" problems has strong implications even for

"nonconvex" problems. Certain existence theorems of Cesari and Olech are thereby extended. Furthermore, growth properties of the minimum value function are derived from growth conditions on the Lagrangian functional being minimized. A noteworthy feature of this paper, aside from the specific results, is the novel use of concepts and approaches of convex analysis in a "nonconvex" setting.

The paper "Saddle-points of Hamiltonian Systems in Convex Problems of Lagrange" [5] studies a class of control problems where one wants to optimize over an infinite time interval. This is related to the way in which certain dynamical systems evolve over time towards an "ideal" state. An application to mathematical economics suggested the results, which are actually valid for Lagrange problems of quite general type.

An invited talk at the IFIP conference on techniques of optimization (organized by A.V. Balakrishnan in Los Angeles, October 1971) led to the expository paper "Dual Problems of Optimal Control" [6], to be published in the proceedings of the conference. This explains in a few pages the principal facts uncovered to date in this line of research. One of the goals, of course, is to open the way for applying duality-based computational methods of convex programming to various control problems.

The paper "Convex Integral Functionals and Duality" [2] corresponds to an invited lecture at the symposium on nonlinear functional analysis sponsored by the Math Research Center (Madison) in April 1971. It sets forth new results on a topic of fundamental importance not only to our control theory work, but also to other work in optimization, particularly stochastic programming. By describing the precise form of the conjugate of an integral functional (such as the expectation of a cost function relative to certain random variables), one is able to reap formulas for the subgradients of the functional, which enter into the necessary and sufficient conditions for an extremum. Compactness properties, useful in proving the existence of limits of minimizing sequences, may also be obtained. In this paper, supplied with an extensive bibliography, the chief contribution is to generalize previous results to the case of functionals whose basic vector argument is infinite-dimensional. Thus tools are provided for extending the control theory work to systems whose states are represented as points in an infinite-dimensional space. Such systems occur, for example, when dynamics are described by a partial differential equation, or when random influences on an ordinary control problem are considered. In the same connection the paper [10] may be mentioned. This was published officially in 1971, although written under an earlier grant. The latter paper furnishes, among other things, new methods for attacking general problems of stochastic optimization. These will be exploited in future research.

The dissertation "Minimax problems, Saddle-functions and Duality" [7] was written by Mr. Lynn McLinden, who was supported as a Research Assistant. McLinden currently holds a two-year research appointment at the Math. Research Center, Madison. The dissertation investigates systematically for the first time the ways of constructing new saddle functions (concave-convex functions) from given ones. It uses this to build a duality theory for problems of minimax type. The results were not easy to achieve. Since saddle functions and minimax problems play a fundamental role in all of optimization theory, it is likely that diverse applications will arise. Similar motivation prompted the paper [9], which too appeared in 1971, although written earlier. This paper extends the modern form of minimax theory to infinite-dimensional problems.

R. T. Rockafellar
Seattle, March 1972

Papers written by R. T. Rockafellar under this grant:

[1] "New applications of duality in convex programming," to appear in the proceedings of the Fourth International Conference on Probability, Brasov, Romania, September 1971.

[2] "Convex integral functionals and duality," in Contributions to Nonlinear Functional Analysis, E. Zarantonello (editor), Academic Press, 1971, 215-235.

[3] "State constraints in convex control problems of Bolza," to appear in S.I.A.M. Journal on Control in 1972.

[4] "Optimal arcs and the minimum value function in problems of Lagrange", submitted to Transactions of the Amer. Math. Soc.

[5] "Saddle points of Hamiltonian systems in convex problems of Lagrange," submitted to J. Optimization Theory and Applications.

[6] "Dual problems of optimal control," to appear in Techniques of Optimization, A.V. Balakrishnan (editor), Academic Press, 1972.

Dissertation written by L. McLinden under this grant:

[7] "Minimax problems, saddle-functions and duality", University of Washington, May 1971 (112 pages).

Papers written by R. T. Rockafellar under preceding AFOSR grant but published in 1971:

[8] "Ordinary convex programs with no duality gap," J. Opt. Theory Appl. 7(1971), 143-148.

[9] "Saddlepoints in convex analysis," in Differential Games and Related Topics, H.W. Kuhn and G.P. Szego (editors), North Holland, 1971, 109-128.

[10] "Integrals which are convex functionals, II", Pacific J. Math. 38 (December 1971).

[11] "Existence and duality theorems for convex problems of Bolza," Trans. Amer. Math. Soc. 159(1971), 1-40.